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ABSTRACT

An analysis is presented of the stressed state of a thin infinite plate with a nearly circular hole with a time-variable pressure applied to the edges of the hole. A solution is obtained using a modification of the "perturbed boundary shape" method, proposed by Savin and Guz' for the solution of static problems of stress concentration near noncircular holes in elastic shells. Using this method, the problem is reduced to the solution of two Helmholtz equations. It is shown that at certain frequencies the stress concentration is higher by 15 to 20 percent than in the static case.

A solution is presented for the state of stress in a thin infinite plate /133* with a hole whose edges are subjected to a pressure varying according to the law $e^{-i\omega t}$. It is assumed that the contour Γ of the hole is close to a circle in the sense that the /134 function $z=\omega(\zeta)$, which carries out the conformal transformation of an infinite plane ζ having a hole consisting of a single circle to the infinite plane z

* Numbers given in margin indicate pagination in original foreign text.

with a hole Γ, has the form

$$\omega(\zeta) = a_0(\zeta + \varepsilon\zeta^{-N}), \tag{1}$$

where $z = re^{i\theta}$; $\zeta = \varrho e^{i\gamma}$; a_0 is the radius of the circular hole which is approached by the contour Γ.

With appropriate values of N and ε we obtain an elliptic, triangular or square hole (ref. 2). The method of the perturbed boundary (ref. 1) is used in a form proposed in reference 3 for solving static problems associated with stress concentration near noncircular holes in elastic shells.

We introduce the dimensionless quantities

$$\left.\begin{aligned} &\bar{r} = \frac{r}{a_0}; \quad \bar{t} = \frac{c_2 t}{a_0}; \quad \bar{u} = \frac{Eu}{2(1+\nu)a_0\sigma_0}; \quad \bar{v} = \frac{Ev}{2(1+\nu)a_0\sigma_0}; \\ &\bar{\sigma}_n = \frac{\sigma_n}{\sigma_0}; \quad \bar{\sigma}_s = \frac{\sigma_s}{\sigma_0}; \quad \bar{\tau}_{ns} = \frac{\tau_{ns}}{\sigma_0}; \quad c_2 = \sqrt{\frac{G}{\mu}}; \quad \bar{\omega} = \frac{a_0\omega}{c_2}, \end{aligned}\right\} \tag{2}$$

where r is the radial coordinate; t is time; u and v are the radial and tangential displacement, respectively; E, G, and ν are, respectively, the Young's modulus, the shear modulus, and the Poisson ratio; c_2 is the velocity of transverse waves; σ_0 is the amplitude of the pressure applied at the contour; σ_n, σ_s, τ_{ns} are the components of the state of stress; μ is the density; ω is the angular frequency.

The problem is reduced to two Helmholtz equations

$$\nabla^2\varphi + \xi^2\omega^2\varphi = 0; \quad \nabla^2\psi + \omega^2\psi = 0, \tag{3}$$

with the additional boundary conditions

$$\sigma_n|_\Gamma = -1; \quad \tau_{ns}|_\Gamma = 0. \tag{4}$$

Here φ and ψ are, respectively, the potentials of longitudinal and transverse waves which are associated with the displacement vector by means of the following relationship

$$\vec{U}\{u, v\} = \vec{\nabla}\varphi + \vec{\nabla}\times\vec{k}\psi,$$

where $\vec{k}$ is the unit vector normal to the plane of the plate; $\xi^2 = \frac{1-\nu}{2}$.

The conditions of radiation must be added to conditions (4).

Below we use only designations introduced by equations (2). Therefore the bars over the letters are omitted. We obtained the following expressions from equation (1) for an elliptic hole with semiaxes a and b.

$$N = 1; \qquad \varepsilon = \frac{a-b}{a+b}; \qquad r = \varrho\sqrt{1 + 2\varepsilon\varrho^{-2}\cos 2\gamma + \varepsilon^2\varrho^{-4}};$$
$$\theta = \operatorname{arctg}\left(\frac{\varrho - \varepsilon\varrho^{-1}}{\varrho + \varepsilon\varrho^{-1}}\operatorname{tg}\gamma\right); \qquad e^{i\alpha} = \frac{\overline{\omega(\zeta)}\,\zeta\omega'(\zeta)}{|\overline{\omega(\zeta)}|\,|\zeta|\,|\omega'(\zeta)|},$$

where α is the angle between the radial direction and the normal to the lines in the z plane representing the transformation of the lines $\rho = const$ in the plane ζ.

Let us expand r, θ, e^{ia} in power series of ε. If Φ designates φ or Ψ, we can obtain the following expansions /135

$$\Phi(r,\theta) = \Phi(\varrho,\gamma) + \varepsilon\left(\frac{\cos 2\gamma}{\varrho}\frac{\partial}{\partial\varrho} - \frac{\sin 2\gamma}{\varrho^2}\frac{\partial}{\partial\gamma}\right)\Phi(\varrho,\gamma) +$$
$$+ \varepsilon^2\left(\frac{1-\cos 4\gamma}{4\varrho^3}\frac{\partial}{\partial\varrho} + \frac{\sin 4\gamma}{2\varrho^4}\frac{\partial}{\partial\gamma} + \frac{1+\cos 4\gamma}{4\varrho^2}\frac{\partial^2}{\partial\varrho^2} -\right.$$
$$\left. - \frac{\sin 4\gamma}{2\varrho^3}\frac{\partial^2}{\partial\varrho\partial\gamma} + \frac{1-\cos 4\gamma}{4\varrho^4}\frac{\partial^2}{\partial\gamma^2}\right)\Phi(\varrho,\gamma).$$

Let us represent the components of the state of stress and of the functions $\varphi(r,\theta)$, $\psi(r,\theta)$ by means of the following series ε

$$\varphi(r,\theta) = \sum_{j=0}^{\infty}\varepsilon^j\varphi_j(r,\theta); \qquad \psi(r,\theta) = \sum_{j=0}^{\infty}\varepsilon^j\psi_j(r,\theta); \tag{5}$$
$$\sigma_n(r,\theta) = \sum_{j=0}^{\infty}\varepsilon^j\sigma_n^{(j)}(r,\theta); \ldots$$

To determine σ_n, σ_s, τ_{ns} in terms of the components of stress in the polar coordinates we make use of the following equations

$$\begin{aligned} \sigma_n &= \sigma_r \cos^2\alpha + \sigma_\theta \sin^2\alpha + 2\tau_{r\theta}\sin\alpha\cos\alpha; \\ \sigma_s &= \sigma_r \sin^2\alpha + \sigma_\theta \cos^2\alpha - 2\tau_{r\theta}\sin\alpha\cos\alpha; \\ \tau_{ns} &= (\sigma_\theta - \sigma_r)\sin\alpha\cos\alpha + \tau_{r\theta}(\cos^2\alpha - \sin^2\alpha). \end{aligned} \tag{6}$$

Here

$$\begin{aligned} \sigma_r &= -\nu\omega^2\varphi + 2\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial^2\psi}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial\psi}{\partial\theta}\right); \\ \sigma_\theta &= -\omega^2\varphi - 2\left(\frac{\partial^2\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial^2\psi}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial\psi}{\partial\theta}\right); \\ \tau_{r\theta} &= -\omega^2\psi + 2\left(\frac{1}{r}\frac{\partial^2\varphi}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial\varphi}{\partial\theta} - \frac{\partial^2\psi}{\partial r^2}\right). \end{aligned} \tag{7}$$

Substituting these expansions into (3), (7), and (6), and collecting the coefficients for each power of ε, we obtain equations for the j-th approximation

$$\nabla^2\varphi_j + \xi^2\omega^2\varphi_j = 0; \qquad \nabla^2\psi_j + \omega^2\psi_j = 0 \tag{8}$$

whose solution is

$$\begin{aligned} \varphi_j &= \sum_{n=0}^{\infty}\left[A_n^{(j)}H_n^{(1)}(\omega\xi r)\cos n\theta + B_n^{(j)}H_n^{(1)}(\omega\xi r)\sin n\theta\right]; \\ \psi_j &= \sum_{n=0}^{\infty}\left[C_n^{(j)}H_n^{(1)}(\omega r)\cos n\theta + D_n^{(j)}H_n^{(1)}(\omega r)\sin n\theta\right], \end{aligned}$$

where $H_n^{(1)}$ is the Hankel function of the first kind.

The boundary conditions have the form

$$\sigma_n^{(j)}|_\Gamma = f_1(\gamma); \qquad \tau_{ns}^{(j)}|_\Gamma = f_2(\gamma). \tag{9}$$

In equations (9) the right part is determined from the preceding /136
approximations, while the operators φ_j, ψ_j in the left part coincide with the corresponding operators for determining stresses using the polar coordinates ρ, γ.

The resulting expressions for the stresses with an accuracy up to ε^3 have the following form

$$\sigma_n(r,\theta) = e^{-i\omega t}\{\sigma_n^{(0)}(\varrho,\omega) + \varepsilon\sigma_n^{(1)}(\varrho,\omega)\cos 2\gamma + \varepsilon^2[\sigma_{n0}^{(2)}(\varrho,\omega) + \sigma_n^{(2)}(\varrho,\omega)\cos 4\gamma]\};$$

$$\tau_{ns}(r,\theta) = e^{-i\omega t}[\varepsilon\tau^{(1)}(\varrho,\omega)\sin 2\gamma + \varepsilon^2\tau^{(2)}(\varrho,\omega)\sin 4\gamma];$$

$$\sigma_s(r,\theta) = e^{-i\omega t}\{\sigma_s^{(0)}(\varrho,\omega) + \varepsilon\sigma_s^{(1)}(\varrho,\omega)\cos 2\gamma + \varepsilon^2[\sigma_{s0}^{(2)}(\varrho,\omega) + \sigma_s^{(2)}(\varrho,\omega)\cos 4\gamma]\}. \qquad (10)$$

TABLE 1

ϱ	$\omega = 1$		$\omega = 2$		$\omega = 3$	
	CIRCLE $\sigma_{n_{max}}$	ELLIPSE $\sigma_{n_{max}}$	CIRCLE $\sigma_{n_{max}}$	ELLIPSE $\sigma_{n_{max}}$	CIRCLE $\sigma_{n_{max}}$	ELLIPSE $\sigma_{n_{max}}$
1.0	1.00	[illegible]	1,00	1,00	[illegible]	[illegible]
1.2	0,79	[illegible]	[illegible]	0,87	[illegible]	[illegible]
1.4	[illegible]	[illegible]	[illegible]	0,80	[illegible]	0.81
1.6	[illegible]	[illegible]	[illegible]	0,76	[illegible]	0.78
1.8	[illegible]	[illegible]	0.75	0.71	0.78	[illegible]
2.0	0,53	0.49	0.71	0,68	0.75	[illegible]
2.5	0,45	[illegible]	0,64	0,62	0,67	[illegible]
3.0	0.43	0.42	0.59	0.57	0.62	0.53
3.5	0.40	0.36	0.55	0.54	0.57	0.49
4.0	0.38	0.32	0.51	0.49	0.54	0.43

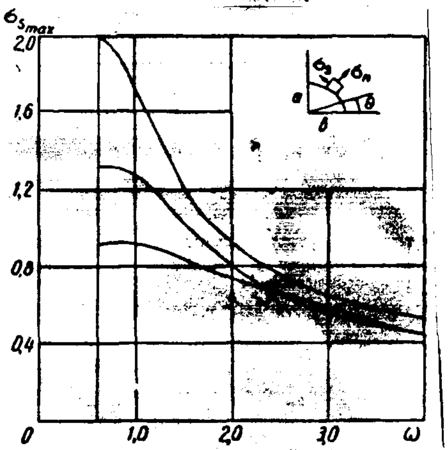

Figure.

The real parts R in (10) give the stresses for t=0, while the imaginary parts I give the solutions for $t=\frac{T}{4}$, $T=\frac{2\pi}{\omega}$. The absolute values $\sqrt{R^2+I^2}$ express the maximum stresses.

If we let ω tend to zero in (10) and make use of the asymptotic properties of cylindrical functions, we obtain expressions in the limit which coincide with the expansion of the stresses corresponding to the static problem in powers of ε. This situation makes it possible to establish the convergence of the solution obtained. Thus, for the example cited below, the error in determining σ_θ along the contour of the hole, when $\omega \to 0$, is less than 1.5 percent.

The numerical results are obtained for $\varepsilon = 1/7$, which corresponds to a/b = 4/3. Calculations were carried out for $\nu = 0.28$. Table 1 shows the values $\sigma_{n_{max}}$ as a function of ρ when $\omega = 1, 2, 3$. Table 2 shows the variation in σ_s along the contour of the hole. The corresponding values of r and θ are determined by (5).

The drawing shows the variation in the maximum stress σ_s at the contour of the hole as a function of the frequency when θ = 0, π/2. In the static case we have when $\varepsilon = 0$ (circular hole) $\sigma_s = 1.0$;

when $\varepsilon = \frac{1}{7}$, $\theta = \frac{\pi}{2}$ $\sigma_s = 0.35$;

when $\varepsilon = \frac{1}{7}$, $\theta = 0$ $\sigma_s = 1.65$.

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TABLE 2

		$\gamma=\frac{\pi}{12}$	$\gamma=\frac{\pi}{6}$	$\gamma=\frac{\pi}{4}$	$\gamma=\frac{\pi}{3}$	$\gamma=\frac{5\pi}{12}$	Circular hole
$\omega=1$	$\mathrm{Re}\,\sigma_s^{(2)}$	0,991	0,914	0,848	0,783	0,764	0,894
	$\mathrm{Im}\,\sigma_s^{(2)}$	1,322	1,117	0,894	0,670	0,549	0,920
$\omega=2$	$\mathrm{Re}\,\sigma_s^{(2)}$	0,071	0,002	—0,014	0,000	0,015	0,009
	$\mathrm{Im}\,\sigma_s^{(2)}$	0,898	0,816	0,767	0,717	0,726	0,816
$\omega=3$	$\mathrm{Re}\,\sigma_s^{(2)}$	—0,057	—0,082	—0,017	—0,212	—0,283	—0,181
	$\mathrm{Im}\,\sigma_s^{(2)}$	0,613	0,539	0,507	0,475	0,500	0,544
$\omega=4$	$\mathrm{Re}\,\sigma_s^{(2)}$	—0,114	—0,160	—0,224	—0,287	—0,333	—0,232
	$\mathrm{Im}\,\sigma_s^{(2)}$	0,524	0,469	0,403	0,336	0,294	0,400

The middle curve is constructed for a circular hole, while the upper and lower curves are for an elliptical hole with $\theta = \pi/2$ and $\theta = 0$.

Thus, as we can see from the figure, for some values of the frequency the stress concentration is greater than 15-20 percent than in the static case.

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